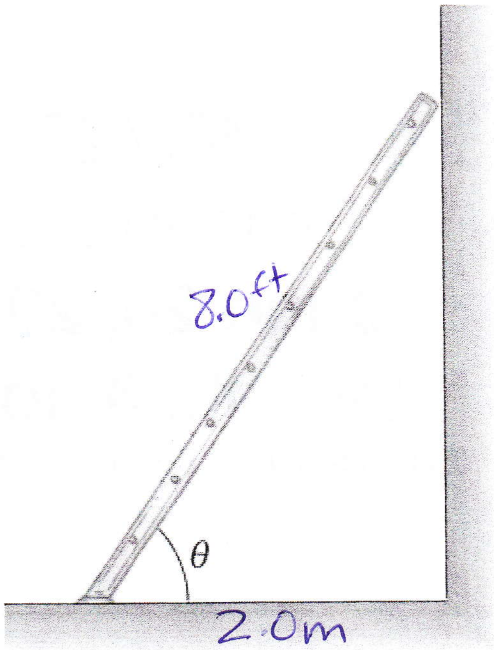
A) Trigonometry

A1) An 8.0 m long ladder is leaning against a fence.
The base of the ladder is 2.0 m from the fence.
Determine the angle the ladder makes with the ground.



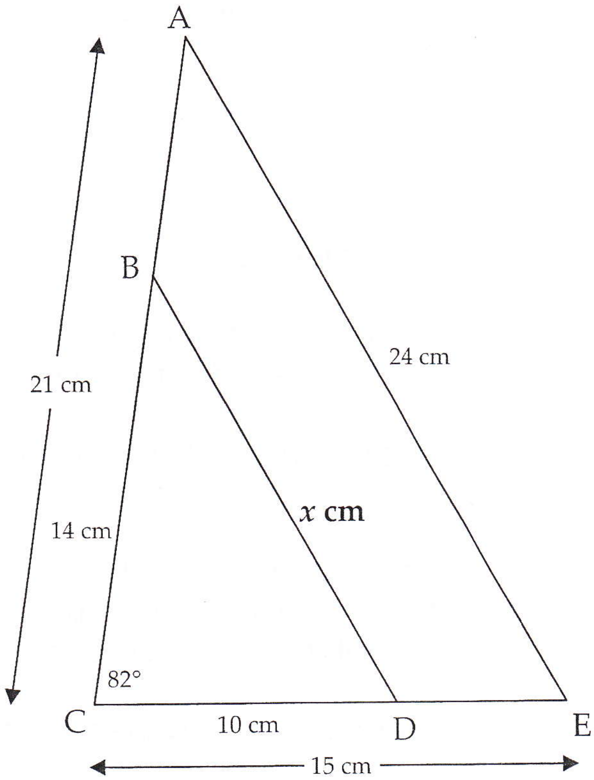
$$\cos\theta = \frac{2.0}{8.0}$$

$$\theta = \cos^{-1}\left(\frac{2.0}{8.0}\right)$$

$$\theta \doteq 75.5^\circ$$

∴ the angle the ladder makes with the ground is 75.5°

A2) **Determine** the value of x. Round to the nearest centimetre.
Verify your answer using another method.



$$\frac{AC}{BC} = \frac{EC}{DC}$$

$$\frac{21}{14} = \frac{15}{10} = \boxed{\frac{3}{2}}$$

$\angle ACE = \angle BCD$ (shared angle)

∴ $\triangle ACE \sim \triangle BCD$ by SAS~

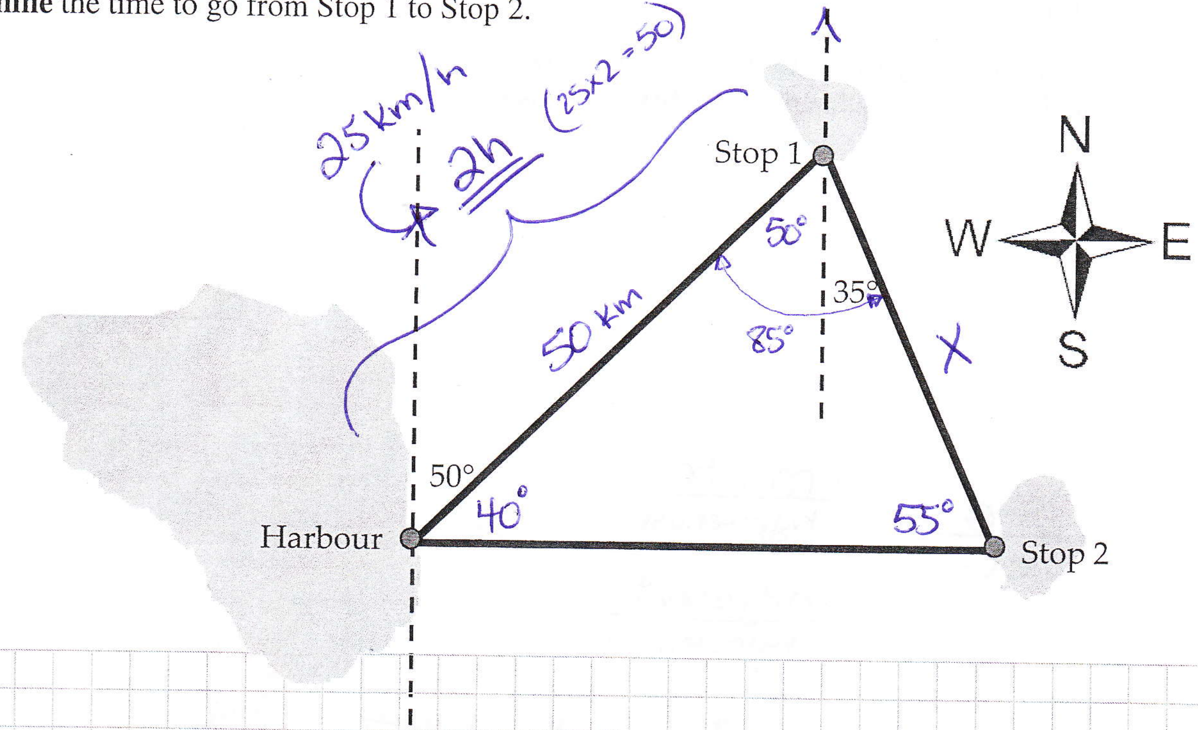
$$\frac{x}{24} = \frac{10}{15}$$

$$x = \frac{10 \times 24}{15}$$

$$x = 16$$

∴ the value of x is 16cm.

A3) A cruise ship, whose path is shown in the diagram below, travels at 25 km/h. It leaves the harbour and travels for two hours before stopping at Stop 1. Later, the ship heads towards Stop 2 which is due east of its starting point. **Determine** the time to go from Stop 1 to Stop 2.



$$\frac{x}{\sin 40^\circ} = \frac{50}{\sin 55^\circ}$$

$$x = \frac{50 \sin 40^\circ}{\sin 55^\circ}$$

$$x \doteq 39.2$$

$$\frac{25\text{ km}}{1\text{ h}} = \frac{39.2\text{ km}}{x}$$

Flip

$$\frac{x}{39.2\text{km}} = \frac{1\text{ h}}{25\text{km}}$$

$$x = \frac{1\text{ h} \times 39.2\cancel{\text{km}}}{25\cancel{\text{km}}}$$

$$x \doteq 1.57$$

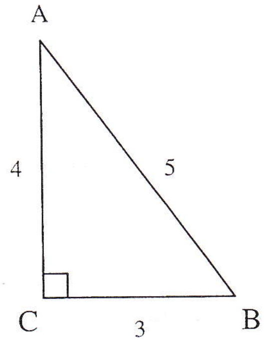
∴ it takes 1.57 hours to go from Stop 1 to Stop 2

A4) Azra's **tan** key on her calculator is broken .

Azra's brother claims that she doesn't need a **tan** key,

she just needs to calculate $\sin\theta \div \cos\theta$.

a) **Verify** that this property works for the triangle below.
b) Do you think it works for all angles? **Justify** your answer.



$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{\text{opposite}}{\text{hypotenuse}}}{\frac{\text{adjacent}}{\text{hypotenuse}}}$$

$$= \frac{\text{opposite}}{\cancel{\text{hypotenuse}}} \times \frac{\cancel{\text{hypotenuse}}}{\text{adjacent}}$$

hypotenuse cancel out

$$= \left.\frac{\text{opposite}}{\text{adjacent}}\right\} \text{equal } \tan\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{4}{5}}{\frac{3}{5}}$$

$$= \frac{4}{5} \times \frac{5}{3}$$

$$= \frac{20}{15}$$

Reduce

$$= \frac{4}{3}$$

$$\tan\theta = \frac{4}{3}$$

equals the same as $\tan\theta$

B) Analytic Geometry

B1) Two continuous linear relations are defined below.
Determine the point of intersection of the lines defined by these relations.

Relation 1:
$y = -3x + 12$

Relation 2:

Run / Rise

x	y
-2	-7
-1	-5
0	-3
1	-1

y-intercept

① $y = -3x + 12$
② $y = 2x - 3$

Substitution

$-3x + 12 = 2x - 3$
$-3x - 2x = -3 - 12$
$\frac{-5x}{-5} = \frac{-15}{-5}$
$x = 3$

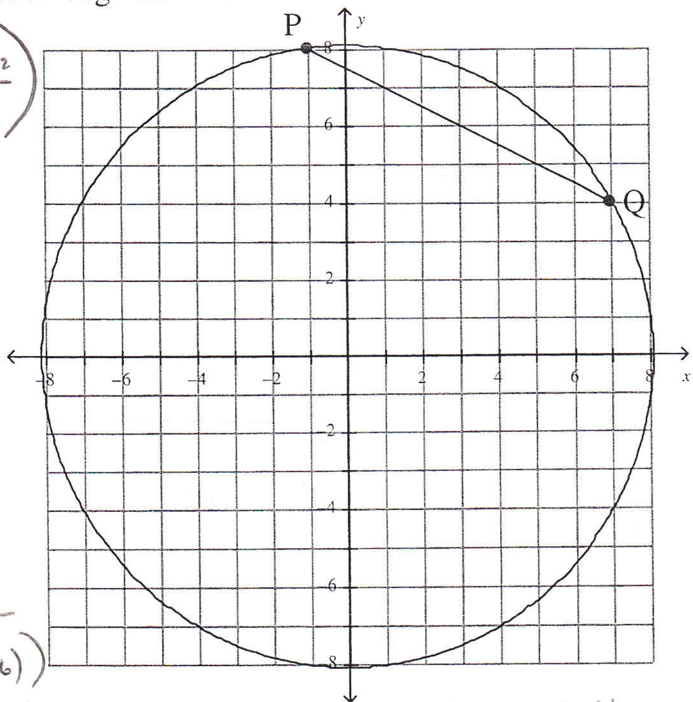
∴ the POI is $(3, 3)$

Slope $= \frac{\text{Rise}}{\text{Rune}}$
$= \frac{2}{1}$

Substitute $x=3$ into equation

$y = -3x + 12$
$y = -3(3) + 12$
$y = -9 + 12$
$y = 3$

B2) The points P(–1, 8) and Q(7, 4) are on the circle with equation $x^2 + y^2 = 65$.
Verify that the perpendicular bisector of PQ passes through the centre of the circle.



$\text{Slope}_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{4-(8)}{7-(-1)}$
$= \frac{-4}{8}$
$= -\frac{1}{2}$

∴ the slope of the perpendicular bisector is $\boxed{2}$

$MP_{PQ} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$\left(\frac{-1+7}{2}, \frac{8+4}{2}\right)$
$\left(\frac{6}{2}, \frac{12}{2}\right)$
$(3, 6)$

Equation of right bisector

$y = 2x + b$ (sub (3,6))
$6 = 2(3) + b$
$6 = 6 + b$
$b = 6 - 6$
$b = 0$

∴ the equation of the right bisector is $y = 2x$
(0,0) satisfies the equation.

B3) Jack works at both the Athena Souvlaki Stop and the Bytown Grill.
One week he worked for 30 hours at Athena and 10 hours at Bytown and earned $510.
Another week he worked for 20 hours at Athena and 30 hours at Bytown and earned $690.
Jack can only work 23 hours next week.
Determine the maximum amount of money Jack can earn.

1st week earnings

① $30x + 10y = 510$

2nd week earnings

② $20x + 30y = 690$

let x represent wage at Athena
let y represent wage at Bytown

Elimination

① $30x + 10y = 510 \quad \times 3$
② $20x + 30y = 690$

subtract

New ① → $90x + 30y = 1530$
② $-\;20x + 30y = 690$

$\frac{70x}{70} = \frac{840}{70}$

$\boxed{x = 12}$

∴ Jack earns #12 per hour at Athena and #15 per hour at Bytown.

sub x=12 into ① to solve y

$30x + 10y = 510$
$30(12) + 10y = 510$
$360 + 10y = 510$
$10y = 510 - 360$
$\frac{10y}{10} = \frac{150}{10}$
$\boxed{y = 15}$

☆ Jack should work all 23 hours at Bytown (#15) and he will earn #345

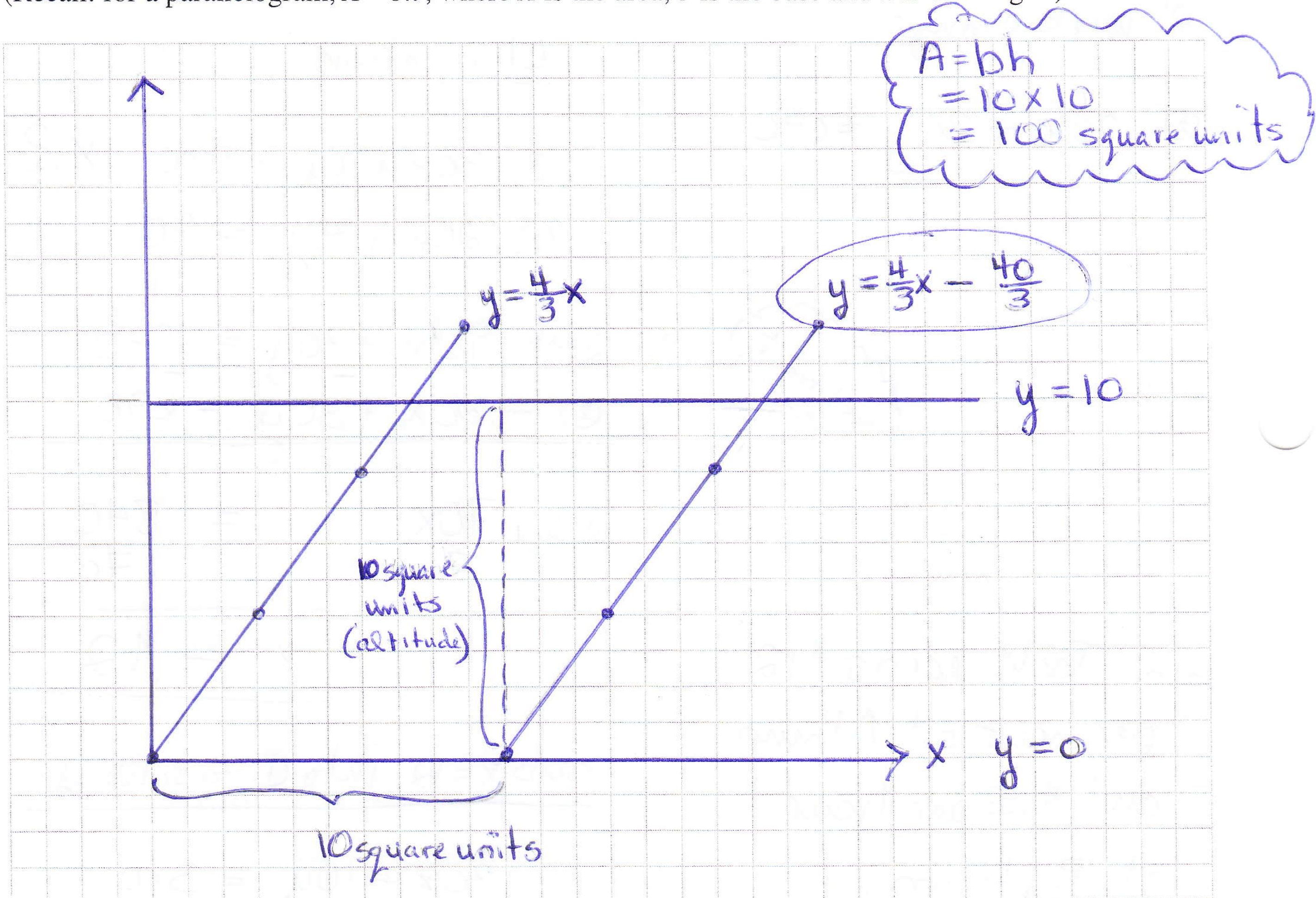
B4) Parallelograms can be created by the line $y = \frac{4}{3}x$ and three other lines.

a) **Explain** how properties of parallelograms can be used to determine equations of the three other lines.

b) Consider a parallelogram with one side contained by $y = \frac{4}{3}x$.

The parallelogram has an area of 100 square units.
Determine one possible set of three equations containing the other sides.
Justify your answer.

(Recall: for a parallelogram, $A = bh$, where A is the area, b is the base and h is the height.)



Solve for b

$y = \frac{4}{3}x + b$

$0 = \frac{4}{3}(10) + b$ sub (10,0)

$-\frac{40}{3} = b$

$b = -\frac{40}{3}$

∴ the four equation for my solution are

$y = \frac{4}{3}x$

$y = \frac{4}{3}x - \frac{40}{3}$

$y = 10$

$y = 0$

C) Quadratic Relations

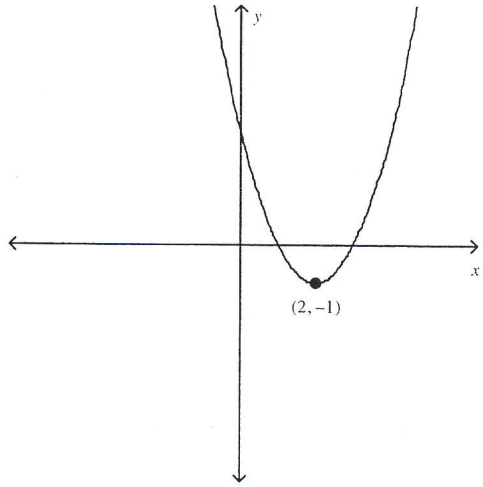
C1) The parabola with equation $y = x^2$ is translated so that its vertex is at (2, –1). **State** an equation of the new parabola.

$y=(x-2)^2-1$ ← Best answer

(OR other possible answers)

$y = (x-1)(x-3)$

$y = x^2-4x+3$



C2) A rocket is launched from the top of a very tall building.
The flight of the rocket can be modeled by $h = -5t^2 + 30t + 80$, where h is the height of the rocket in meters relative to ground, t seconds after being launched.
Determine the maximum height of the rocket.

$h = -5t^2+30t+80$

$= -5(t^2-6t)+80$

$= -5(t^2-6t+9-9)+80$

$= -5[(t-3)^2-9]+80$

$= -5(t-3)^2+45+80$

$= -5(t-3)^2+125$

OR

$x = \frac{-b}{2a}$

$= \frac{-(30)}{2(-5)}$

$= \frac{-30}{-10}$

$= 3$

sub x=3

$= -5(3)^2+30(3)+80$

$= 125$

$y=-5(x-3)^2+125$

C3) **Determine** the zeros of the quadratic relation with equation $y = -2x^2 - 16x - 24$.

$y = -2x^2-16x-24$

$0 = -2(x^2+8x+12)$

$0 = -2(x+6)(x+2)$

$x+6=0$ $\boxed{x=-6}$ OR $x+2=0$ $\boxed{x=-2}$

OR

P: 48
S: -12-4=-1
I: -12, -4

$0 = -2x^2-16x-24$

$= -2x^2-4x-12x-24$

$= -2x(x+2)-12(x+2)$

$= (-2x-12)(x+2)$

$= -2(x+6)(x+2)$

C4) The table below shows data for average time on Facebook by age

Age	Average number of minutes per day on Facebook
13	54
14	96
15	126
16	144
17	150
18	144

Vertex

1st

$96-54=42$
$126-96=30$
$144-126=18$
$150-144=6$
$144-150=-6$

2nd

$30-42=-12$
$18-30=-12$
$6-18=-12$
$-6-6=-12$

2nd differences constant "Quadratic"

a) **Determine** an equation which could be used to predict Facebook usage for other ages.

b) Could this model be used to predict average Facebook use for all Facebook users? **Justify** your answer.

$y=a(x-17)^2+150$

Sub (13,54) into equation to solve a

$54=a(13-17)^2+150$

$54-150=16a$

$\frac{-96}{16}=\frac{16a}{16}$

$a=-6$

An equation is $y=-6(x-17)^2+150$

table of values

Age	Average
11	-66
12	0
13	54
14	96
15	126
16	144
17	150
18	144
19	126
20	96
21	54
22	0

b) If this equation was correct for all ages, it would mean that there would be negative usage for some ages (less than 12 and greater than 22) but this is clearly not true.

C5) Quadratic relations may have 2, 1 or 0 zeros.
Using a variety of forms of equations and representations, write examples of each case and **justify** your choices.

1) If the quadratic relation has two zeros,

a) the parabola has its vertex above the x-axis and opens down
"a" < 0 and "k" > 0

eg $y = -x^2 + 1$ opens down, vertex (0,1) and zeros -1 & 1

b) the parabola has its vertex below the x-axis & opens up (a > 0 & k < 0)

eg $y = x^2 - 1$ opens up, vertex (0,-1) and zeros -1 & 1

c) the equation can be written in factored form with two distinct zeros

eg $y = (x-1)(x-2)$

2) If the quadratic relation has one zero,

a) the parabola has it vertex on the x-axis (k = 0)

eg $y = x^2$ vertex at (0,0), having one zero of 0.

b) factored form eg $y = (x-2)(x-2)$ single x-intercept at 2

3) If the quadratic has no zeros,

a) the parabola has its vertex above the x-axis & open up ie a > 0 & K > 0

eg $y = x^2 + 1$ vertex (0,1) and has no zeros.

b) the parabola has its vertex below the x-axis & opens down ie a < 0 & K < 0

eg $y = -x^2 - 1$ vertex (0,-1) & has no zeros.